Formal Definition

A CA is a 4-tuple (A, V, S, R) where

- A is the space of cells. In practice A is the ensemble of coordinates $\vec{r}$ labelling the cells. Typically, $\vec{r} = (i, j)$ in two dimensions, where i and j are integers.

- V is a neighborhood stencil. For each $\vec{r} \in A$, one defines $V(\vec{r})$ as the set $V(\vec{r}) = \{\vec{r} + \vec{v}_0, \vec{r} + v_1, ..., \vec{r} + \vec{v}_z\}$ where $\vec{v}_0 = 0$ and z is the lattice coordination number.

- S is the set of possible states for each cell. The state of cell $\vec{r}$ is denoted $s(\vec{r}) \in S$.

- R is the CA rule. It is a function giving the new state $s_{t+1}(\vec{r})$ as a function of the states $s_t(\vec{r}\,')$ at iteration t, where $\vec{r}\,'$ belongs to the neighborhood $V(\vec{r})$ of $\vec{r}$: $s_{t+1} = R(\{s_t(V(\vec{r})\})$

Global dynamics

We can also consider that the rule acts on the whole cell configuration c_t. The CA dynamics is then

$$c_{t+1} = \mathcal{R}(c_t)$$

Neighborhood

- von Neumann
- Moore
- Margolus (a dynamic neighborhood)
- ...

Rule Calculation

- On the fly, with an algebaric expression (see parity rule).
- With a pre-computed lookup table (which rapidely becomes huge...)

Some special cases

- Linear rules (in the sense of XOR)
- Totalistic rules (R is function of the sum of the neighborhood states.
- ....

Boundary Conditions

- Periodic
- Reflexive
- Adiabatic
- fixed
- With specified behavior after rule computation
- ...

Extended CA's

- non-uniform (or heterogeneous) CA
- Asynchronous CA (various degrees of asynchronicity)
- Stochastic CA
- CA with memory (e.g. time-reversible rule)
- Propagation-Interaction model

All these extensions actually fit with the basic definition if the state space is enlarged and the rule is adjusted.

Some interesting questions

- Wolfram's rule: 1D, radius r number of state per cell k. Complexity classes (in the sense of complex and dynamical systems), fixed points, limit cycles, attractors
- Inverse problem: density task, synchronization task,...(search the rule space with evolutionnary computing methods)

Applications

- Trafic models. Fundamental diagram, examples
- Lattice gases: discrete physics models